A note on the sensitivity to yaw of a hot-wire anemometer

By C. A. G. WEBSTER

Department of the Mechanics of Fluids, University of Manchester

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It is often supposed that the mean velocity indicated by a yawed hot-wire anemometer is the component of the stream velocity normal to the wire. Experiments have been performed which show that a better approximation is obtained by taking the velocity as the resultant of the normal component of the mean stream and 0·2 times the component parallel to the wire.

Introduction

The measurement, by hot-wire anemometers, of the cross-components of a field of turbulence requires the use of these instruments inclined at angles other than normal to the incident mean flow. Generally a pair of wires is provided, inclined at approximately 45° to and on either side of the stream direction. Frequently the sensitivity to yaw of such inclined wires is obtained by a direct calibration, but to obtain a calibration with sufficient accuracy is not an easy task and on occasions conditions render it impossible. Thus it would seem desirable to have a formula, even if only empirical, which could lead to an estimate of the sensitivity of a hot-wire anemometer to yaw, independently of a calibration made specifically for this parameter.

Studies of the basic properties of hot-wire anemometers have been carried out with the wires normal to the stream, e.g. Collis & Williams (1959), and from the parameters of and the current flowing through such a wire, it is possible to calculate the velocity of the mean stream with some accuracy. In the case of an inclined wire some relationship is required to express the mean stream velocity in terms of the wire angle and the apparent velocity calculated directly from the wire parameters. Thus if ϕ is the angle of incidence of the wire, i.e. the angle between the normal to the wire and the stream direction, and $U(\phi)$ the velocity calculated from the wire parameters when it is set at angle ϕ, we wish to find an empirical function $k(\phi)$ such that

$$U(\phi) = U(0)\,k(\phi), \tag{1}$$

where $U(0)$ is the calculated velocity at zero incidence.

It has been shown by Prandtl (1946), Struminsky (1946) and Sears (1948) that, for laminar flows past an infinitely long inclined cylinder, the components normal to the cylinder are independent of that parallel to the axis. It follows that the rate of heat loss per unit length depends only on the normal component of velocity and hence $k(\phi) = \cos\phi$. Subsequently it has often been supposed that this result may be applied directly to wires of finite length. Schubauer & Klebanoff (1946) carried out experiments to test the validity of the 'cosine-law' and concluded that it held for finite wires, the observed deviations from the law being attributed to the ageing of their wire. Wyatt (1955) also conducted an experiment and, detecting no significant departures, decided that the 'cosine-law' was applicable at least up to $\phi = 30°$.

By contrast Hinze (1959) puts forward an alternative expression for $k(\phi)$ such that

$$U^2(\phi) = U^2(0)(\cos^2\phi + a^2\sin^2\phi), \tag{2}$$

where the factor a takes a value between 0·1 and 0·3 depending on the magnitude of the velocity (decreasing as the velocity is increased). In the limiting case of very long wires, a should also tend to zero so that one reverts to the form $k(\phi) = \cos\phi$.

Whilst the treatment for an infinite wire is quite simple there seemed to be little hope of successfully producing a theoretical solution for the problem of a finite wire. An experimental approach appeared more promising and accordingly a small experiment was started to try to establish whether the factor a of Hinze's formula was indeed non-zero and, if so, to determine values for the parameter under various conditions.

The experimental arrangement

The experiments were carried out in a wind tunnel which had been designed to have a low level of turbulence. The wires were mounted horizontally between the pair of vertical supports of a wire holder and the whole unit plugged into the top of a vertical tube which passed through the floor of the tunnel and was capable of unlimited rotation about its own vertical axis. The hot wire was viewed from above through a telescope fitted with a protractor eyepiece, the latter being used to measure the angle made by the etched portion of the wire with respect to some fixed datum. The layout is sketched in figure 1. Horizontal mounting of the wire was chosen so that the effect of free convection, which is most probably negligible in any case, would be independent of the angle of orientation. The determination of wire angles by direct observation, rather than by measuring the rotation of the support rod, was preferred since a wire is liable to be deflected with respect to its supports under the aerodynamic loading of the airstream.

The wires used for the experiments were all made from Wollaston wire with a nominal core diameter of $2{\cdot}5 \times 10^{-3}$ mm, although a photomicrograph of a cross-section of a sample of the wire revealed the core to be of somewhat elliptical shape with major and minor axes of $2{\cdot}7 \times 10^{-3}$ mm and $1{\cdot}8 \times 10^{-3}$ mm respectively, a fact which may have contributed to the scatter of the results of the experiment. The sensitive elements of the anemometers were obtained in the usual manner by etching away the silver sheath of the Wollaston wire in a fine jet of acid; by the use of suitable nozzles, etchings as short as 0·19 mm could be obtained. With these very short wires the central section was often noticeably out of alignment with respect to the unetched portions so that, even without aerodynamic deformation, the optical method of angle determination was essential. Wires were made covering a range of length from the lower limit of 0·19 mm up to about 3·2 mm but with a preference for lengths around 1 mm, which is a very convenient size for turbulence measurements.

The hot wires were used in one arm of a bridge circuit and operated in the constant-current manner. Having been placed in the tunnel, each wire was first adjusted to be normal to the air stream and with the wire fixed in this position the out-of-balance current of the bridge calibrated against a set of different wind speeds, these, in turn, being determined from pressure measurements. Having calibrated the wire, the tunnel wind speed was fixed at some suitable value (for the most part of the order of 6·2 m/sec but at about 4 m/sec for a small group of six runs) and the bridge out-of-balance readings taken over a range of angles of incidence.

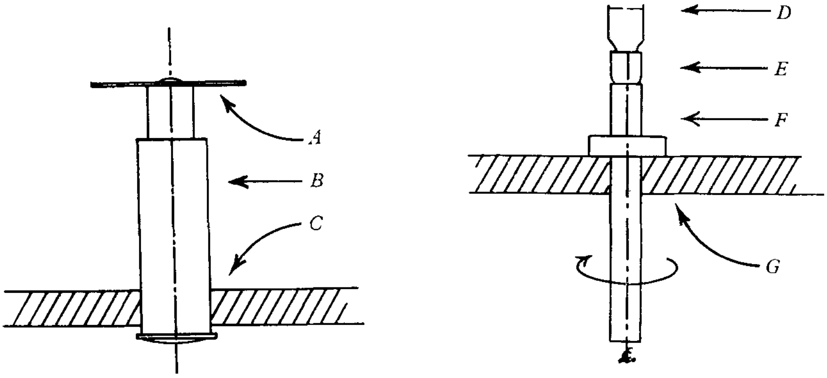

FIGURE 1. Schematic diagram of the apparatus. The telescope B, fitted with a protractor eye piece A and mounted through the wind-tunnel roof C, is used to view the hot-wire D the holder E of which is plugged into a rotatable support F, passing through G the floor of the tunnel.

A current of 20 mA was used to heat the wires. The effective wire temperature, that is to say the temperature obtained directly from the wire's resistance, is *inter alia* a function of wire length and wind speed. Typical effective temperature excesses above ambient are 120 °C for a 0·5 mm wire and 200 °C for a 2·0 mm wire at 6·2 m/sec.

The experiments were only carried out on clear days of good visibility when changes of wire calibration due to contamination by air-borne dust particles would be very slight. An examination of the wires through a microscope at the end of a run generally showed them to be quite clean but, in any case, a dirty wire usually leads to an underestimate of the velocity at the wire and would, in this experiment, reduce the value of a.

To minimize any systematic error arising from asymmetry of the wire with respect to its supports, the readings were spread fairly uniformly over 360°, whilst avoiding angles of incidence in the ranges 65° to 115° and 245° to 295° over which the sensitive element of the wire might have been in the wake of a wire support. The effect of the wire supports on the flow for angles outside these ranges has been investigated on a potential theory basis and it was found that, to the first-order of the correction when calculating a, the effect of the acceleration of the flow cancelled that arising from the tilting of the streamlines. In view of the scattered nature of the values of a finally obtained there seemed to be nothing to be gained by making a more refined estimate of the correction term. The reduction of the experimental results was carried out by first using the wire calibration to convert, for each angle, the observed out-of-balance currents to their equivalent velocities and then finding, for each run, the best value of a (in the sense of least squares) by fitting the data to equation (2). The 'as calculated' values of a are set out in table 1 below, together with the corresponding wire lengths and wind speeds; graphs are also shown in which a is plotted against wire length (figure 2) and wind speed (figure 3).

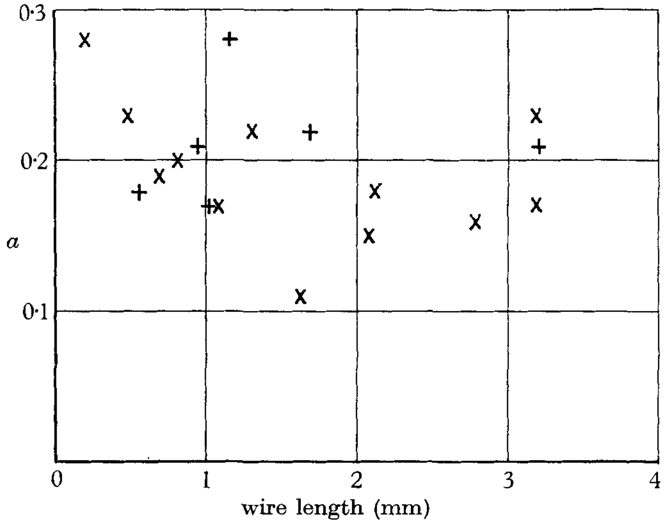

FIGURE 2. The parameter a plotted against wire length. Points + belong to the 4 m/sec group and points × to the 6·2 m/sec group.

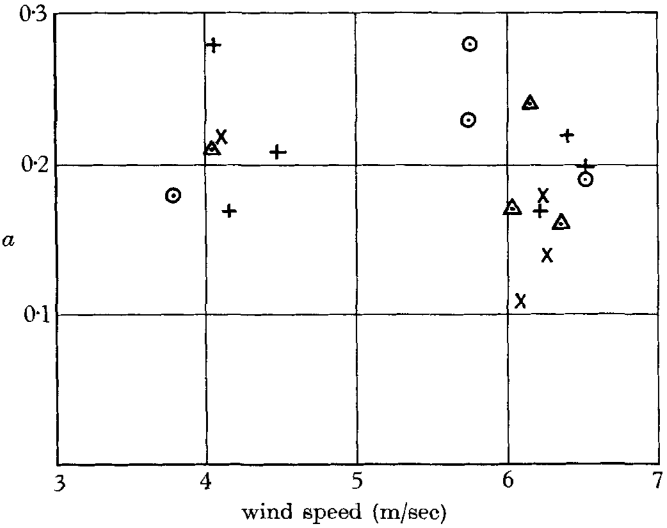

FIGURE 3. The parameter a plotted against wind-tunnel speed. Points ⊙ are for wire lengths in the range 0 to 0·75 mm, + in the range 0·75 to 1·5 mm, × in the range 1·5 to 2·25 mm and △ those in excess of 2·25 mm.

Wire length (mm)	Length/diam. ratio	Tunnel speed (m/sec)	a
0·55	249	3·78	0·18
0·94	426	4·48	0·21
1·04	472	4·15	0·17
1·15	522	4·05	0·28
1·68	762	4·10	0·22
3·21	1456	4·04	0·21
0·19	86	5·75	0·28
0·47	213	5·74	0·23
0·68	308	6·51	0·19
0·81	367	6·51	0·20
1·07	485	6·20	0·17
1·30	590	6·39	0·22
1·62	735	6·07	0·11
2·07	939	6·26	0·14
2·12	962	6·23	0·18
2·79	1266	6·35	0·16
3·18	1442	6·02	0·17
3·18	1442	6·15	0·24

The length-to-diameter ratios are based on a diameter of $2{\cdot}2 \times 10^{-3}$ mm, the geometrical mean of the lengths of the major and minor axes of the wire's section.

TABLE 1

Conclusions

Although the value of a obtained in these experiments is scattered there seems to be little doubt that it is non-zero. The results show no systematic tendency to depend on wire length and, although experimental difficulties precluded the taking of observations over a very wide range of tunnel speeds, they also appear to indicate at most only a weak variation with velocity (see figures 2 and 3).

Taking all the runs together a mean value of

$$a = 0{\cdot}20 \pm 0{\cdot}01 \tag{3}$$

may be obtained. Using (3), equation (2) becomes

$$U^2(\phi) = U^2(0)\,(\cos^2\phi + 0{\cdot}04\sin^2\phi), \tag{4}$$

leading to values of $dU(\phi)/d\phi$ which are less than those given by the 'cosine-law' by 5 % at 30°, 6 % at 45° and 9 % at 60°.

The lack of variation of a with velocity is not unexpected but its apparent independence of wire length is a little surprising, especially in view of the wide range of wire lengths covered, namely 0·19 to 3·21 mm, corresponding to length-to-diameter ratios from 86:1 to 1456:1, for the longest of which a tendency to zero might have been anticipated, so as to accord with the theoretically predicted result of zero for infinitely long wires.

No convincing explanation of the phenomenon has been found; it is just possible that it might be associated with the elliptical section of the wires, since any twisting of them would lead to a departure from two-dimensional flow, although this seems unlikely to be significant.

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